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A note on the density wave model of the galaxy

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In this paper the general gas dynamical equations have been solved in the wave form and the general dispersion relation has been deduced. This dispersion relation has been used with simplifying assumptions plausible for special regions of the Galaxy, and results obtained have been shown to be able to interpret some observed dynamical behaviours as well as the distributional property of the gas in those special regions. For example, the analysis has yielded the interpretation of (a) the absence of any wave-pattern in the central region of the Galaxy, (b) the large-scale deviation of the gas from the galactic plane in the outer regions of the Galaxy and (c) probably, the large-scale outflow of gas in the central region, as well as the large outward motion of the 3kpc arm. The analysis further indicates that in the solar neighbourhood the rotation curve of the Galaxy may possess a local maximum.

1. INTRODUCTION

The formation of spiral arms in disk galaxies and their maintenance over a long period of time, inspite of the presence of differential rotation in these galaxies, have been intriguing questions to the astronomers for a long time Lindblad suggested many years ago that the phenomena might be explained in terms of the density waves Lindbald's original suggestion has recently been developed and put on a firm mathematical basis by Lin & Shu (1964, 1966) and by Lin (1967a, b). Basu (1971) has used the model to explain some observed phenomena in the solar neighbourhood and also to draw some plausible inferences of a general nature. Basu & Roy (1972) have extended the model to the inner region of the Galaxy and tried to interpret, on the basis of their calculations, some of the observed dynamical behaviours of the gas in the central region. All these works suggest that the density wave model of the spiral structure of the Galaxy helps to explain many peculiar observed phenomena regarding the motion and distribution of the gas in it. In the present paper we have derived the general dispersion relation from the wave solution of the linnearized three-dimensional gas dynamical equations and deduced some conclusions, on the basis of the analysis of the dispersion relation under particular conditions.

2. LINEARIZED EQUATIONS DERIVED

The three-dimensional gas dynamical equations in cylindrical coordinates (r, θ, z), for a gas rotating about the z-axis, are

$$\frac{\partial \rho}{\partial t}+\frac{1}{r}\frac{\partial}{\partial r}(r\rho u)+\frac{1}{r}\frac{\partial}{\partial \theta}(\rho v)+\frac{\partial}{\partial z}(\rho w)=0 \qquad \ldots \quad (1)$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial r}+\frac{v}{r}\frac{\partial u}{\partial \theta}+w\frac{\partial u}{\partial z}-\frac{v^2}{r}=-\frac{\partial \phi}{\partial r} \quad . \quad (2)$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial r}+\frac{v}{r}\frac{\partial v}{\partial \theta}+w\frac{\partial v}{\partial z}+\frac{uv}{r}=-\frac{1}{r}\frac{\partial \phi}{\partial \theta} \quad . \quad (3)$$

$$\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial r}+\frac{v}{r}\frac{\partial w}{\partial \theta}+w\frac{\partial w}{\partial z}=-\frac{\partial \phi}{\partial z} \quad . \quad (4)$$

together with the Poisson's equation

$$\frac{\partial^2 \phi}{\partial r^2}+\frac{1}{r}\frac{\partial \phi}{\partial r}+\frac{1}{r^2}\frac{\partial^2 \phi}{\partial \theta^2}+\frac{\partial^2 \phi}{\partial z^2}=4\pi G\rho. \quad . \quad (5)$$

where ϕ is the gravitational potential and G the constant of gravitation. Before perturbation, let the density, potential and velocity components be given by,

$$\begin{aligned} \rho &= \rho_0 \\ \phi &= \phi_0 \\ u &= 0 \\ v &= r\Omega(r) \\ w &= 0 \end{aligned} \quad \ldots \quad (6)$$

where $\Omega(r)$ is the angular velocity at a distance r from the center. After perturbation these become

$$\begin{aligned} \rho &= \rho_0+\rho' \\ \phi &= \phi_0+\phi' \\ u &= u' \\ v &= r\Omega(r)+v' \\ w &= w' \end{aligned} \quad \ldots \quad (7)$$

where the dashed quantities are those due to perturbation. Using (7) in (1)-(5) and linearizing in perturbtion quantities, we get the following set of linearized equations :

$$\frac{\partial \rho'}{\partial t}+\Omega\frac{\partial \rho'}{\partial \theta}+\rho_0\frac{\partial u'}{\partial r}+\frac{\rho_0}{r}\frac{\partial v'}{\partial \theta}+\rho_0\frac{\partial w'}{\partial z}=0 \quad \ldots \quad (8)$$

$$\frac{\partial u'}{\partial t}+\Omega\frac{\partial u'}{\partial \theta}-2\Omega v'=-\frac{\partial \phi'}{\partial r} \quad \ldots \quad (9)$$

$$\frac{\partial v'}{\partial t}+\Omega\frac{\partial v'}{\partial \theta}+\frac{k^2}{2\Omega}u'=-\frac{1}{r}\frac{\partial \phi'}{\partial \theta} \quad \ldots \quad (10)$$

$$\frac{\partial w'}{\partial t}+\Omega\frac{\partial w'}{\partial \theta} = -\frac{\partial \phi'}{\partial z} \quad \cdots \quad (11)$$

$$\frac{\partial^2\phi'}{\partial r^2}+\frac{1}{r}\frac{\partial \phi'}{\partial r}+\frac{1}{r^2}\frac{\partial^2\phi'}{\partial \theta^2}+\frac{\partial^2\phi'}{\partial z^2} = 4\pi G\rho' \quad \cdots \quad (12)$$

where k is the epicyclic frequency defined by

$$k^2 = 4\Omega^2\left[1+\frac{r}{2\Omega}\frac{d\Omega}{dr}\right] \quad \cdots \quad (13)$$

The third term in equation (8) is actually

$$\frac{1}{r}\frac{\partial}{\partial r}(r\rho_0 u') = \frac{1}{r}\left[r\rho_0\frac{\partial u'}{\partial r}+u'\frac{\partial}{\partial r}(r\rho_0)\right]$$

Now, u' is a small quantity and $r\rho_0$ changes slowly with r compared to the rapid change of u' with r. Thus the second term appearing in the third brackets above is small compared to the first term. We therefore have

$$\frac{1}{r}\frac{\partial}{\partial r}(r\rho_0 u') \sim \rho_0\frac{\partial u'}{\partial r}.$$

3. Solution of equations and dispersion relations

Let us assume the following wave solutions for the equations (1)-(5). (Lin & Shu 1964, Basu 1971, for two-dimensional case)

$$\left.\begin{aligned} u' &\equiv \exp i[\omega t-n\theta+k_1r+l_1z] \\ v' &\equiv \exp i[\omega t-n\theta+k_2r+l_2z] \\ w' &\equiv \exp i[\omega t-n\theta+k_3r+l_3z] \\ \rho' &\equiv \exp i[wt-n\theta+k_4r+l_4z] \\ \phi' &\equiv \exp i[\omega t-n\theta+k_4r+l_4z] \end{aligned}\right\} \quad \cdots \quad (14)$$

where ω is the wave frequecy, k_t, n and l_t are the wave numbers in the directions of the coordinates (r, θ, z), respectively n essentially represents the number of spirals and we shall use $n = 2$ Using (14) and (8)-(5), the solutions are given by

$$u' = \frac{rD_1k_4+2\,in\Omega}{rD}\phi' \quad (15)$$

$$v' = -\frac{2n\Omega D_1-irk_4k^2}{2r\Omega D}\phi \quad (16)$$

$$\frac{l_4\varphi}{D_1} \quad (17)$$

$$\rho' = -\frac{k_4^2 - i\frac{k_4}{r} + \frac{n^2}{r^2} + l_4^2}{4\pi G}\phi' \qquad \dots \quad (18)$$

where

$$\left.\begin{aligned} D &= k^2-(\omega-n\Omega)^2 \neq 0 \\ D_1 &= \omega-n\Omega \neq 0 \end{aligned}\right\} \qquad \dots \quad (19)$$

and

Using (15)-(18) in equation (8) we obtain the dispersion relation as

$$\frac{k_4^2}{D_2} - \left(\frac{k_1}{D} + \frac{i}{D_2 r} - \frac{ink^2}{2\Omega D D_1 r}\right)k_4 - \frac{2in\Omega k_1}{DD_1 r} + \frac{l_4^2}{D_2} + \frac{l_3 l_4}{D_1^2} = \frac{n^2}{r^2}\left(\frac{1}{D} - \frac{1}{D^2}\right), \quad \dots \quad (20)$$

where $D_2 = 4\pi G\rho_0$ The dispersion relation (20) characterizes the propagation of density waves through the medium of interstellar gas. We note that the dispersion relation (20) involves the wave numbers in radial, tangential and axial directions implying that the density waves propagating in radial, tangential and the axial directions are *coupled*. The dispersion relation (20) also indicates that the waves do not propagate in the region of the Galaxy where D and D_1 as given by (19) vanish. The same conclusion was arrived at by Lin (1967a) from a quite different approach He called the region where $D \sim 0$ as the principal part of Galaxy.

4. Dispersion relation in special cases

We shall now study the dispersion relation (20) under special conditions and analyse the results in order to draw plausible inferences.

Case 1. We consider the dispersion relation at large distances from the centre of the Galaxy, that is, as $r \to \infty$. The relation (20) becomes

$$\frac{k_4^2}{D^2} - \frac{k_1 k_4}{D} + \frac{l_4^2}{D_2} + \frac{l_3 l_4}{D_1^2} = 0 \qquad \dots \quad (21)$$

If we now *assume* that $k_1 = k_4 = l_3 = l_4$, then the relation (21) simplifies to

$$\frac{2}{D_2} + \frac{1}{D_1^2} = \frac{1}{D} \qquad \dots \quad (22a)$$

or written explicitly this become

$$\frac{\omega}{n} = \Omega \pm \frac{1}{n\sqrt{2}}[(k^2-D_2)\pm\sqrt{K^4+D_2^2}]^{\frac{1}{2}}. \qquad \dots \quad (22b)$$

which shows that the relation between the angular velocity of the pattern ω/n and that of the material Ω is essentially dependent on the epicyclic frequency and the basic density of matter If we further assume that the density waves propagate along the galactic plane, then $l_3 = l_4 = 0$. With $k_1 = k_4$, we now get

$$\omega = n\Omega \pm (k^2 - 4\pi G\rho_0)^{\frac{1}{2}}. \qquad \dots \quad (23)$$

This shows that there are two modes of propagation of density waves along the galactic plane both of which will be stable in time if

$$k^2 > 4\pi G\rho_0$$

is satisfied. The modes will both be unstable, however if

$$k^2 < 4\pi G\rho_0 \qquad \dots \quad (24)$$

Thus at large distances from the galactic centre, the stability of density waves essentially depends on the mutual relation between the epicyclic frequency and the basic density of matter.

We can now study the behaviour of the density waves at large distances from the galactic centre, using the numerical values of the parameters involved, as have been given by Basu (1971, 1972) and by Basu & Roy (1972) The last column of table 1 shows that $4\pi G\rho_0 \gg k^2$, at all distances from the galactic centre We are concerned with the outer region of the Galaxy for which the inequality (24) has been derived Since this holds in the outer regions, the density waves propagating there along the galactic disk will be unstable. The relation (23) reduces to the form

$$\omega = \alpha \pm i\beta. \qquad \dots \quad (25)$$

where both α and β are positive. Relation (25) shows that under the conditions we have specified, there will be two modes of propagation of the density waves, one of which will decay while the other will be amp.ified. This amplificztion of density waves in the outer parts of the Galaxy may be physically associated with the large distortion of the galactic disk observed by many authors (Kerr 1957, Burke 1957), in these regions. The important observed facts of the distorted region are that it contains a number of spiral features as if the spiral-arm condensations are superposed and that the gas layer greatly deviates from the mean plane of the Galaxy (Kerr & Westerhout 1965) Although a number of explanations have been suggested for these phenomena by various authors, we like to emphasize that both of the above features find fairly satisfactory explanation as being the manifestation of the amplification of density waves.

Case 2. We consider the dispersion relation very close to the centre, that is, for very small values of r. The relation turns out to be $\omega = n\Omega \pm (k^2 - 4\pi G\rho_0)^{\frac{1}{2}}$ which is the same as (23). But here no restrictions have been made regarding the wave numbers in the tangential and axial directions. The last column of table 1 shows that the relation $4\pi G\rho_0 \gg k^2$ is very well satisfied for small values of r also. Thus in the central region of the Galaxy all density waves will be unstable leading to the diffusion of any spiral pattern that might be generated here. This fact is relevant to observation in the central region, where no spiral pattern has so far been detected.

Case 3. Let us now discuss the nature of the density wave propagating along perpendicular to the galactic disk under the assumption that the radial propagation is not important, that is, $k_\nu = 0$, $\nu = 1, 2, 3, 4$. The dispersion relation (20) then takes the form

$$l_4^2 = \frac{n^2}{r^2}\frac{D_1^2(D_2-D)}{D(D_2+D_1^2)} \tag{26}$$

The relation (26) shows that such density waves do not propagate at large distances from the galactic centre, as well as in the region where

$$k^2-(\omega-n\Omega)^2 = 4\pi G\rho_0. \quad \ldots \quad (27)$$

holds. But columns 5 and 8 of table 1 shows that the relation (27) holds nowhere in the main part of the Galaxy (1-12 kpc). Thus in the main part of the Galaxy the propagation of density waves perpendicular to the galactic plane is important. The velocity of propagation is given by

$$|v_{\rm ph}|_z = \left|\frac{\omega}{\text{Real } l_4}\right| = \frac{r\omega\{k^2-(\omega-n\Omega)^2\}^{\frac{1}{2}}}{n\,|(n\Omega-\omega)|} \quad \ldots \quad (28)$$

The values of $|v_{\rm Ph}|$ for different values of r are plotted in figure 1. We see that the values are very large in 2 5-4 kpc region with a sharp maximum at 3 kpc. The 3 kpc region of the Galaxy should have, therefore, a large-scale disturbance in the axial direction. The validity of this conclusion may be a reality in view of the observed facts of large-scale outflow of gas in the central region and the large motion of the 3 kpc arm. The large-scale outflow of matter over the entire region of the Galaxy may be dynamically related with the large-scale disturbances in the axial direction in the 2.5-4 kpc region, as has been envisaged here.

Case 4. We now consider a more general case of the density waves for which all the wave numbers in the radial and axial directions are equal, that is, $k_1 = l_1 = k_2 = l_2 = k_3 = l_3 = k_4 = l_4$.

The dispersion relation (20) then takes the form

$$k_4^2\left(\frac{2}{D_2}-\frac{1}{D}+\frac{1}{D_1^2}\right)-\frac{ik_4}{r}\left(\frac{1}{D_2}-\frac{nk^2}{2\Omega}\frac{1}{DD_1}+\frac{2n\Omega}{DD_1}\right)$$

$$= \frac{n^2}{r^2}\left(\frac{1}{D}-\frac{1}{D_2}\right) \quad \ldots \quad (29)$$

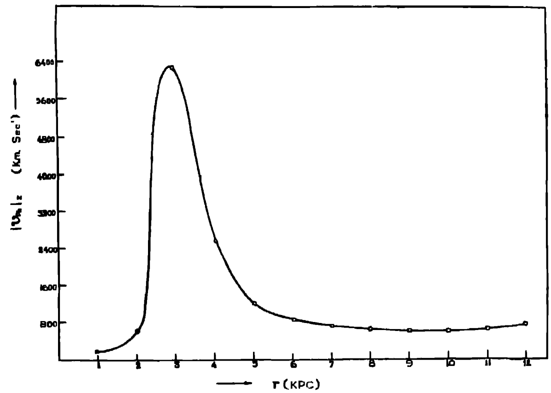

Figure 1. The phase velocity in the z -direction shown against the distance from the galactic centre. The sharp peak in the 3-kpc region indicates an unusual disturbance in the axial direction in this region

Table 1 shows that throughout the region 2-12 kpc of the Galaxy the inequalities $D << D_2$

i.e., $$k^2 << (\omega-n\Omega)^2+4\pi G\rho_0$$

and $$D_1^2 << D,$$

i.e., $$2(\omega-n\Omega)^2 << k^2.$$

are satisfied. Using these inequalities, the dispersion relation (29) becomes

$$\frac{k_4^2}{D_1^2} - ik_4 \frac{n(4\Omega^2-k^2)}{2\Omega r D D_1} = \frac{n^2}{r^2 D} \quad \dots \quad (30)$$

This yields

$$k_4 = \frac{D_1^2}{2r}\left[i\frac{(4\Omega^2-k^2)n}{2\Omega D D_1} \pm \left\{ \frac{4n^2}{DD_1^2} - \frac{(k\Omega^2-k^2)^2 n^2}{4\Omega^4 D^2 D_1} \right\}^{\frac{1}{2}} \right] \quad \dots \quad (31)$$

Relation (31) shows that the density waves cease to propagate where

$$\frac{4n^2}{DD_1^2} < \frac{(4\Omega^2-k^2)^2 n^2}{4\Omega^2 D^2 D_1^2}$$

i.e. where $(4\Omega^2-k^2)^2 > 16\Omega^2 D$ holds.

On the otherhand, in the region where $k^4 < 16\Omega^2 D$ holds, there must be two unstable modes of propagation of density waves, one of which will be damped while the other will be amplified. If, on the otherhand, the wavelengths are such that

$$\frac{k_4}{D_1^2} >> \frac{(4\Omega^2-k^2)n}{2\Omega r D D_1}$$

that is,

$$\lambda_4 << \frac{2\Omega r D}{n D_1(4\Omega^2-k^2)} \qquad \ldots \quad (32)$$

is satisfied, then the dispersion relation (30) simplifies to

$$k_4 = \pm \frac{n(\omega-n\Omega)}{r[k^2-(\omega-n\Omega)^2]^{\frac{1}{2}}} \qquad \ldots \quad (33)$$

This dispersion relation (33) clearly shows that stable modes of density waves whose wavelengths do not exceed certain critical value characterized by the relation (32), can propagate in the region of the Galaxy where the inequality

$$k^2-(\omega-n\Omega)^2 > 0 \qquad \ldots \quad (34)$$

holds. Table 1 shows that this inequality holds throughout the region 1-12 kpc of the Galaxy. In Schmidt's (1965) model, the inequality holds in the region 2-12 kpc. The region of validity of the inequality (34) may be called the principal part of the Galaxy.

The values of the right hand side of (32) are 8.8, 21.4 and 15 kpc respectively, at $r = 2$, 6 and 10 kpc. Thus the modes of density waves in the solar neighbourhood with wavelengths far less than 15 kpc will be stable. The modes of longer wavelength may be stable at the middle region of the Galaxy. Toomre (1964) arrived at similar conclusions from his analysis of the gravitational instability of the *galactic disk*. He, however, has limited his analysis to axisymmetric disturbances.

5. Perturbation of velocity component

Using equations (15)-(18) we get

$$\frac{u'}{\rho'} = -\frac{rk_4(\omega-n\Omega)+2in\Omega}{rD} \cdot \frac{4\pi G}{k_4^2-\frac{ik_4}{r}+\frac{n^2}{r^2}+l_4^2}$$

$$\frac{v'}{\rho'} = \frac{2n\Omega(\omega-n\Omega)-irk^2k_4}{2\Omega r D} \cdot \frac{4\pi G}{k_4^2-\frac{ik_4}{r}+\frac{n^2}{r^2}+l_4^2}.$$

$$\frac{w'}{\rho'} = \frac{l_4}{\omega-n\Omega} \cdot \frac{4\pi G}{k_4^2-\frac{ik_4}{r}+\frac{n^2}{r^2}+l_4^2}.$$

Table 1

r (KPC)	Ω (km sec^{-1} kpc^{-1})	$(\omega - n\Omega)^2$ (km^2 sec^{-2} kpc^{-2})	k^2 (km^2 sec^{-2} kpc^{-2})	$D^{\frac{1}{2}} = [k^2-(\omega-n\Omega)^2]^{\frac{1}{2}}$ km.sec^{-1} kpc^{-1}	ρ_0 M_0pc^{-3}	$D_4 = 4\pi G\rho_0$ (km. sec^{-1} kpc^{-1})^2	$(4\pi G\rho_0 - k^2)^{\frac{1}{2}}$ (km. sec^{-1} kpc^{-1})
1	120 00	25600.00	44100.00	136.00	3 905	218300	417.50
2	80.00	1600.00	17690.00	126.90	1 915	107100	299.00
3	65.00	11.09	11030.00	105 00	1 235	69050	240.80
4	53 25	42.25	7073.00	83 87	973	54420	217.60
5	45.40	116.60	4794 00	68 39	661	36970	179.40
6	39 67	162 40	3406.00	56 89	.505	28240	157.60
7	35.27	180.60	2494.00	48 09	386	21590	138 20
8	31 50	169.00	1824.00	40 68	291	16280	120.20
9	28.11	139.70	1367 00	35.02	212	11850	102.30
10	25.00	100.00	990.60	29.97	145	8108	84.30
11	22.18	63 36	725.70	25.74	099	5536	69.40
12	19.83	40.44	501.80	21.48	069	3859	57.90

$n = 2$
$\omega/2 = \Omega p =$ angular velocity of the pattern; $r\Omega_p =$ constant $= 200$ km.sec^{-1}
$\Omega =$ angular velocity of the material.

To simplify the discussion we restrict ourselves in the region where $k_4^{-1} << r$ holds. Basu (1971) obtained the radial wave number to be π-kpc^{-1} from his discussion of the two-dimensional density waves. So the relation $k_4^{-1} << r$ may be considered to represent the regions at some fair distances from the galactic centre. In these regions we have

$$\left|\frac{u'}{\rho'}\right| = \frac{4\pi G k_4 |(n\Omega-\omega)|}{D\left(k_4^2+\frac{n^2}{r^2}+l_4^2\right)}\left[1+\frac{4n\Omega^2}{r^2(\omega-n\Omega)^2}\right]^{\frac{1}{2}} \quad \ldots \quad (35)$$

$$\left|\frac{v'}{\rho'}\right| = \frac{4\pi n G |(\omega-n\Omega)|}{rD\left(k_4^2+\frac{n^2}{r^2}+l_4^2\right)}\left[1+\frac{k^4 k_4^2 r^2}{4n^2\Omega^2(\omega-n\Omega)^2}\right]^{\frac{1}{2}} \quad \ldots \quad (36)$$

$$\left|\frac{w'}{\rho'}\right| = \frac{4\pi G l_4}{|(\omega-n\Omega)|\left(k_4^2+\frac{n^2}{r^2}+l_4^2\right)} \quad \ldots \quad (37)$$

Since all the parameters involved on the right hand sides of the expressions (35)-(37) are functions of the central distance r, the ratios of the velocity to density perturbations are different at different regions of the Galaxy This dependence is shown in figures 2 and 3. To draw these figures, we have neglected the term n^2/r^2 from the denominators of the relations (35)-(37), since this term is relatively small. The figures show the ratios u' and v' increase steadily with r and that the latter is always greater than the former except at $r = 2$ kpc. This result is consistent with that obtained by Basu & Roy (1972) with their two-dimensional analysis of the density waves. The ratio w'/ρ', on the otherhand, shows curious behaviour. It has a very sharp peak at 3 kpc, then falls rapidly to a minimum at 7 kpc and again steadily rises to high values till the boundary of the region considered. This large perturbation velocity in the axial direction near the boundary may be associated with the observed large deviation of the gas from the galactic plane in this region. We can compare the curves in figure 2, with those in figure 4, where we have plotted $|u'|$, $|v'|$ against r with the assumption that $\rho'/\rho = 0.6$ for gas everywhere in the Galaxy (Basu & Roy 1972) and the values of have been taken from Schmidt's (1965) model. Figure 4 also compares the curves for $|w'/\rho'|$ and $|w'|$. Similar curves will be obtained from the perturbed stellar velocity components on the assumption that $\rho' = 0.1$ for stars. Only the magnitudes of the velocities will be each depressed by a factor of 6. We see that the curve for $|u'|$ steadily falls all the way down from 2 to 12 kpc, while that for v' has a second peak at 10 kpc. This may be regarded as merely a local phenomenon and may be interpreted as the local region possessing a relatively higher angular velocity. The curve for $|w'|$ again shows a very sharp peak at 3 kpc and then falls all the way down to the boundary. The sharp peaks for

$|w'/\rho'|$ and $|w'|$ in figure 4 are consistent with our prediction in the previous section in the region between 2.4-4 kpc from the centre, the Galaxy may have an unusual degree of disturbance in the axial direction.

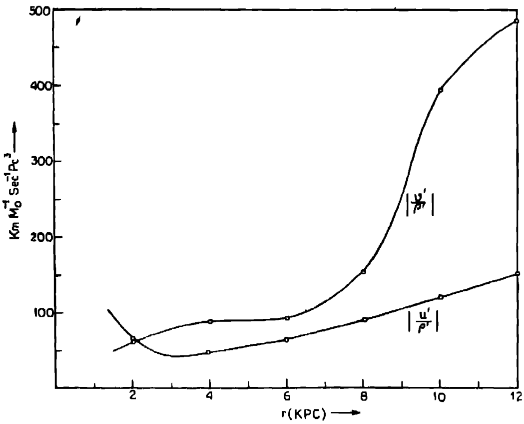

Figure 2. The ratios of perturbed velocity to perturbed density in the radial and tangential directions shown against the distances from the galactic centre.

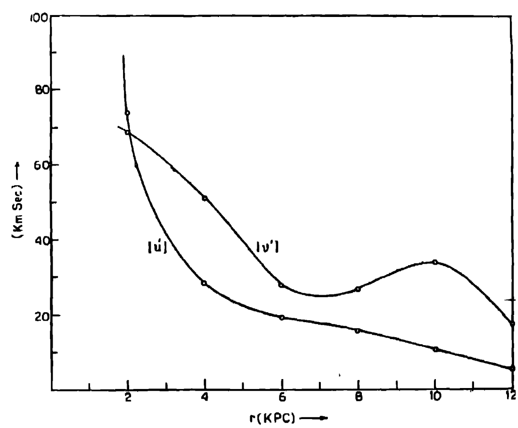

Figure 3. The radial and tangential velocity perturbation shown against the distance from the galactic centre. The peak of v' in 10 kpc region probably signifies a higher rotation of material in the solar neighbourhood.

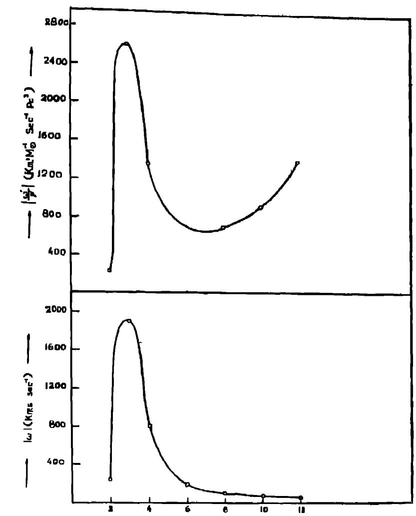

Figure 4 The upper curve is the ratio of the perturbed velocity in the axial direction to the perturbed ensity The lower curve is the perturbed velocity in the axial direction. Both curves show peaks at the 3 kpc region.

6. Summary

We have considered here the density waves of a very genral pattern and how they propagate in different regions of the Galaxy. Usng the values of the various parameters across the entire galactic plane, as have been used earlier by Basu (1971) and by Basu & Roy (1972) we have been able to interpret (a) the absence of any density wave pattern in the central region of the Galaxy, (b) the large-scale deviation of the gas from the galactic plane in its outer regions. We have also tried to interpret the large-scale outflow of gas from the inner regions, and the outward motion of the 3 kpc arm, as dynamically related to the very large-scale disturbances in the axial direction in the 2.5-4 kpc region, as has been suggested from our present analysis. It has also been demonstrated that within the principal part of the Galaxy given by $k^2-(\omega-n\Omega)^2 > 0$, that is within 2-12 kpc region, stable modes of density waves can propagate provided their wavelengths are limited to certain values. The perturbed axial velocity to density ratio curve as well as the perturbed axial velocity curve in figure 4, both confirm the very high degree of axial disturbances in the 2.5-4 kpc region. The perturbed tangential velocity curve suggests that in the solar neighbourhood the rotation curve may have a local maximum.

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